Classe: TS2ET	Date: 13/11/2017	**Type** **Devoir surveillé**
Devoir n°3		
Thème: Transformée de Laplace		

Exercice 1:

Pour chaque cas, tracer la représentation graphique des fonctions données et déterminer la transformée de Laplace de celles-ci.

1°) f(t)=U(t) où U(t) est la fonction échelon unité.

2°) g(t)=U(t-2)

3°) h(t)=t.U(t)

4°) k(t)=4(U(t)-U(t-2))

Exercice 2:

Déterminer les transformées de Laplace des fonctions définies par :

$f_1(t)=2e^{4t}U(t)$

$f_2(t)=(3t-5)U(t)$

$f_3(t)=(t^4-e^{-3t})U(t)$

$f_4(t)=3\cos(7t).U(t)$

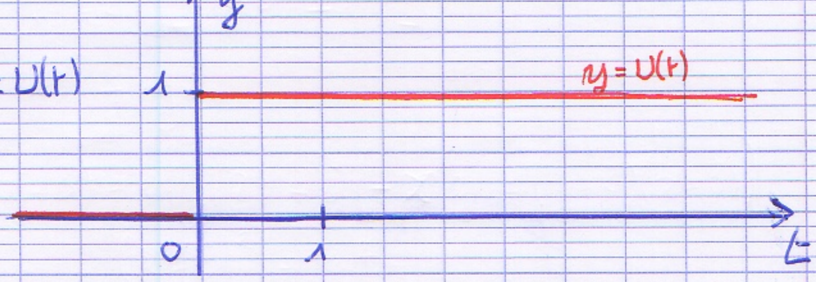
$f_5(t)=t^3e^{-3t}U(t)$

$f_6(t)=e^{3t}\sin(t)U(t)$

Correction

Exercice 1

1°) $f(t) = U(t)$

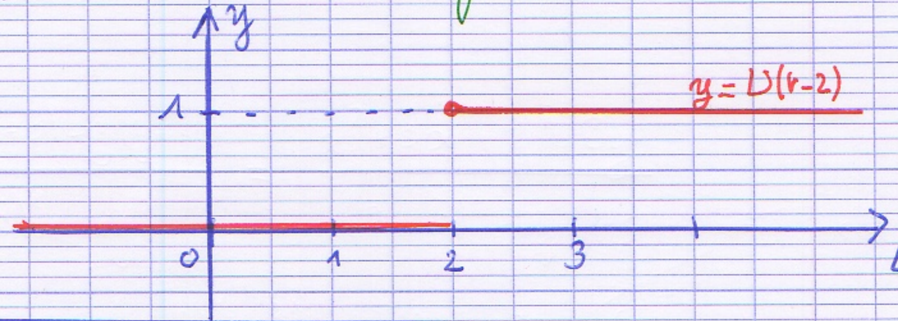


1pt

$$\boxed{\mathcal{L}(f(t)) = \mathcal{L}(U(t)) = \frac{1}{p}}$$

1pt

2°) $g(t) = U(t-2)$ C'est la fonction U avec un retard de 2

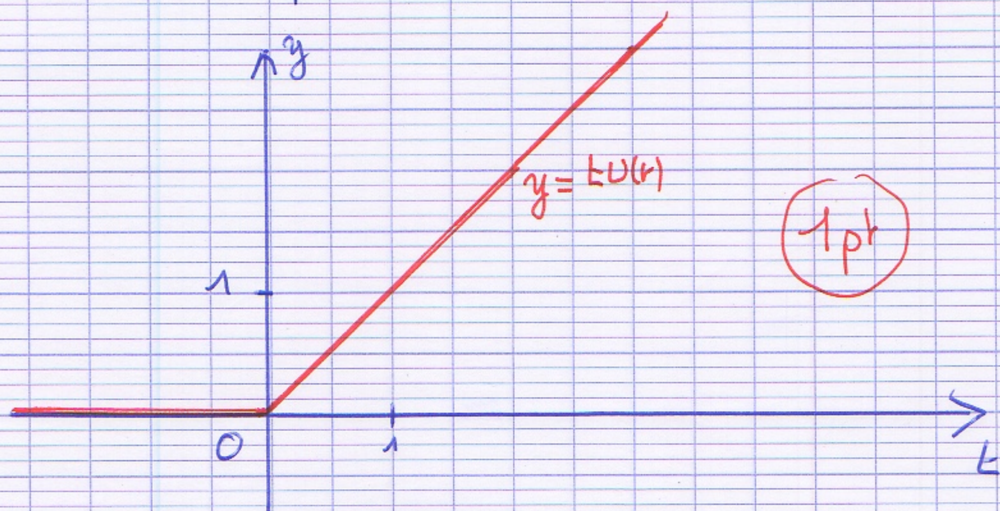


1pt

$$\boxed{\mathcal{L}(g(t)) = \mathcal{L}(U(t-2)) = \frac{1}{p} e^{-2p}}$$

1pt

3°) $h(t) = tU(t) = \begin{cases} t & \text{si } t \geq 0 \text{ car dans ce cas } U(t) = 1 \\ 0 & \text{si } t < 0 \text{ car dans ce cas } U(t) = 0 \end{cases}$

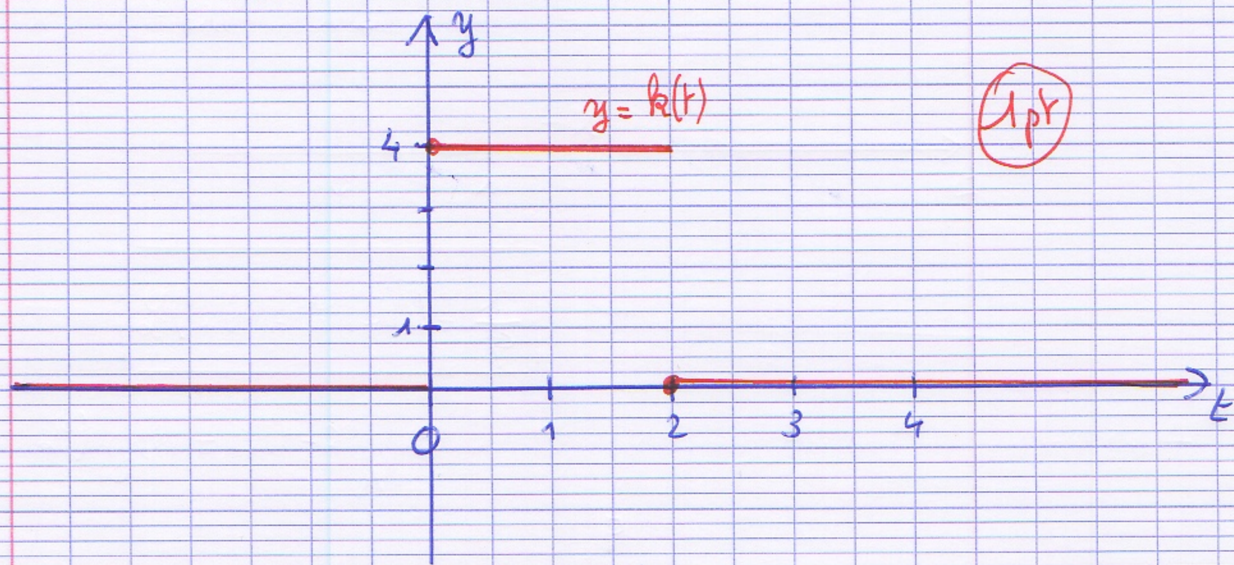


1pt

$$\boxed{\mathcal{L}(h(t)) = \mathcal{L}(tU(t)) = \frac{1}{p^2}}$$

1pt

4°) $k(t) = 4\big(U(t) - U(t-2)\big)$



(1pt)

$$\mathcal{L}(k(t)) = 4\big(\mathcal{L}(U(t)) - \mathcal{L}(U(t-2))\big)$$

$$\boxed{\mathcal{L}(k(t)) = 4\left(\frac{1}{p} - \frac{1}{p}e^{-2p}\right) = \frac{4}{p}\left(1 - e^{-2p}\right)}$$ (1pt)

Exercice 2 $f_1(t) = 2e^{4t}U(t)$

donc $F_1(p) = 2\mathcal{L}(e^{4t}U(t)) = 2\cdot\frac{1}{p-4} = \boxed{\frac{2}{p-4}}$ (2pt)

$f_2(t) = (3t-5)U(t) = 3tU(t) - 5U(t)$

donc $F_2(p) = 3\cdot\frac{1}{p^2} - 5\cdot\frac{1}{p} = \boxed{\frac{3}{p^2} - \frac{5}{p}}$ (2pts)

$f_3(t) = (t^4 - e^{-3t})U(t) = t^4U(t) - e^{-3t}U(t)$

$F_3(p) = \mathcal{L}(t^4U(t)) - \mathcal{L}(e^{-3t}U(t)) = \frac{4!}{p^5} - \frac{1}{p+3} = \boxed{\frac{24}{p^5} - \frac{1}{p+3}}$

(2pts)

$f_4(t) = 3\cos(7t)\,U(t)$

$$F_4(p) = 3 \cdot \frac{p}{p^2+49} = \boxed{\frac{3p}{p^2+49}}$$ (2pts)

$f_5(t) = t^3 e^{-3t}\,U(t)$

$$F_5(p) = \frac{3!}{(p+3)^4} = \boxed{\frac{6}{(p+3)^4}}$$ (2pts)

$f_6(t) = e^{3t}\sin(t)\,U(t)$

$$F_6(p) = \boxed{\frac{1}{(p-3)^2+1}}$$ (2pts)